# The 'bursting' phenomenon in a turbulent boundary layer

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Using a hot wire in a turbulent boundary layer in air, an experimental study has been made of the frequent periods of activity (to be called 'bursts') noticed in a turbulent signal that has been passed through a narrow band-pass filter. Although definitive identification of bursts presents difficulties, it is found that a reasonable characteristic value for the mean interval between such bursts is consistent, at the same Reynolds number, with the mean burst periods measured by Kline *et al.* (1967), using hydrogen-bubble techniques in water. However, data over the wider Reynolds number range covered here show that, even in the wall or inner layer, the mean burst period scales with *outer* rather than inner variables; and that the intervals are distributed according to the log normal law. It is suggested that these 'bursts' are to be identified with the 'spottiness' of Landau & Kolmogorov, and the high-frequency intermittency observed by Batchelor & Townsend. It is also concluded that the dynamics of the energy balance in a turbulent boundary layer can be understood only on the basis of a coupling between the inner and outer layers.

## 1. Introduction

The mechanism of energy production in a turbulent boundary layer, especially in the inner region, has attracted considerable attention in recent years. On the basis of extensive experimental work in a water tunnel, Kline *et al.* (1967) have suggested that the intermittent occurrence of bursts at the edge of the inner layer plays a key role in turbulent energy production. Many characteristics of the bursting phenomenon (such as the rate of occurrence of bursts, their spanwise separation, average velocity, etc.) have been measured by them, and scaled with the basic wall flow parameters  $U_*$  (the friction velocity) and  $\nu$  (the kinematic viscosity). Theoretical models showing an instability in the wall flow have also been proposed by various workers (e.g. Black 1966). On the other hand, several attempts have been made to view the inner layer as in some sense driven by the outer flow (e.g. Schubert & Corcos 1967; Landahl 1967); such analyses, based on linear equations, have explained several features of the observed flow, but have failed to predict Reynolds stresses of the right order.

The experimental investigations of Kline et al. (1967) were all made in water,

using for the most part hydrogen-bubble techniques. In view of the importance of the bursting phenomenon to an understanding of the dynamics of the turbulent boundary layer, it was felt that an independent study in air, using hot-wire techniques, might yield further useful information; the present report describes the results of such a study. As a hot wire can normally make only local measurements, many of the quantities measured by Kline *et al.* involving a visualization of sections of the three-dimensional flow field, are not accessible to a single hotwire probe. Nevertheless, there remain some quantities (like the time interval Tbetween bursts at a point), which can be measured by both techniques. (A set of correlation measurements by Tu & Willmarth (1966) in a boundary layer in air provides us with one valuable point for the mean interval  $\overline{T}$  at a high Reynolds number.) The presence of turbulence in the background makes an accurate estimation of these parameters difficult; and considerable effort was devoted, therefore, to studying the relation between the phenomena noticed in a hot-wire trace and those observed in hydrogen-bubble photographs.

Section 2 describes the experimental set-up; §3 gives the results, along with a comparison with earlier data; and §4 offers a broad interpretation.

### 2. Experimental set up

The experiments were conducted in  $1 \text{ ft} \times 1 \text{ ft}$  low-speed open circuit wind tunnel, driven by a fan on a 10 h.p. induction motor at the downstream end of the tunnel diffuser. Suitable honeycomb and screens were provided ahead of the contraction to obtain uniform flow and to cut down the turbulence level in the test section. The free-stream longitudinal turbulence level in the test section, measured by a hot wire, was found to be 0.3%. At any station along the 8ft long test section, the longitudinal velocity variation is less than 1% of the mean free-stream velocity over most of the cross-section. By the use of two adjustable flaps at the end of the test section, it is possible to control the speed at the test section accurately to 1 % from 5 to 100 ft/sec. All measurements were made on the inner surface of the top wall, which was smooth and polished. Static pressure holes of 0.25 mm diameter were drilled at suitable intervals all along the centreline of the top wall. Rough emery strips were used to hasten and fix transition immediately after the tunnel contraction. Traverse of the pitot or hot wire across the boundary layer was performed by means of a vertical traverse gear indicating on a gauge graduated in 0.01 mm divisions.

The velocity fluctuations of the turbulent flow were registered by a hot wire made of Pt-Rh wire of 0.0001 in. diameter, and nearly 1 mm length. The time constant of such hot wires is approximately 0.2 msec; and the necessary compensation was determined by the well-known square-wave technique. The signal from the hot wire was amplified by an amplifier with compensating circuit incorporated in it. A differentiating circuit, an audio-frequency filter with variable bandwidth and a wave analyzer were used, when necessary, to filter out the unwanted parts of the signal.

The wall stress was measured using either a Preston tube (diameter 0.032 in. calibrated in a two-dimensional channel) or the Clauser plot (Clauser 1956).

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# 3. Results

A direct, 'unprocessed' signal from a hot-wire probe, sensing (say) the longitudinal velocity fluctuation u', does not show anything that can be unambiguously identified as a burst, unless, of course, one is in the outer intermittent region of the boundary layer. A few typical traces, all taken in the inner or wall-similarity region to avoid possible confusion with this outer intermittency, are shown in figures 1(a, b), plate 1. All traces in figure 1(a) were taken at the same Reynolds number; the first is an unprocessed signal, and other traces show the effect of differentiation and of the removal of low frequencies. It is well known in studies of intermittency near the edges of shear flows (e.g. Townsend 1956) that the differentiated signal can strengthen the contrast between quiescent and active periods. However, as figures l(a, b) show, it is the use of filters that brings out most clearly the existence of intermittent periods of considerable activity, which we may, tentatively and for convenience, call 'bursts'. In the earlier stages of this work, we mostly used a variable cut-off high-pass filter (Dawe, type 1462A) in looking for the presence of such bursts; however, after becoming aware of the investigations of Sandborn (1959), we have used a narrow band-width wave analyzer instead (General Radio, type 1554-A). It is clear that if the spectrum of the turbulent signal is falling rapidly at the cut-off frequency (as it must apparently be for 'bursts' to become noticeable), there will not be much difference between the output signals from the high-pass filter and from a wave analyzer with pass band near the cut-off frequency. Our measurements confirmed this, but they also showed that the use of a wave analyzer was sometimes slightly more convenient.

Even from these filtered signals, however, it is clear that the duration or frequency of bursts cannot always be estimated with accuracy, and that further careful processing, of the kind found necessary in intermittency measurements (e.g. Fiedler & Head 1966), may be required before completely reliable values can be obtained. However, certain general trends can be established beyond reasonable doubt, as we shall see below.

The first question to be answered concerns the relation between the phenomena observed in such hot-wire traces and those described by Kline *et al.* (1967) from hydrogen-bubble photographs. This question has been briefly considered earlier by Sandborn (1959); but he made no quantitative comparisons, as he only obtained a measure of the intermittency, whereas the hydrogen-bubble data concern the mean period between bursts. The latter can in principle be measured from hot-wire traces; for, as examination of figure 1 shows, there is no doubt that there are alternate periods of more or less activity. In particular, at relatively low Reynolds numbers (say  $R_{\theta} \simeq 500$ -1000, based on free-stream velocity U and momentum thickness  $\theta$ ), the frequency of the bursts can be counted without much difficulty. Thus, from trace (iv) in figure 1 (a) (where the bursts are identified by an arrow), the mean period  $\overline{T}$  between bursts can be determined to  $\pm 10 \%$ .

It is very gratifying to note that, although the values so obtained for  $\overline{T}$  in the present experiments are different by orders of magnitude from those obtained in the water tunnel by a completely different technique (Kline *et al.* 1967),

	$R_{ heta}$	U (ft sec <sup>-1</sup> )	$ \frac{\nu}{(\mathrm{ft}^2~\mathrm{sec}^{-1})} $	<u>Т</u> (sөс)	$U_*$ (ft sec <sup>-1</sup> )	$\frac{U^2_*\bar{T}}{\nu}$	$\frac{U\overline{T}}{\delta^*}$
Kim et al. (1968) Present measure- ments	666 620	$\begin{array}{c} 0{\cdot}25\\ 17{\cdot}2 \end{array}$	$1.15 \times 10^{-5}$ $1.85 \times 10^{-4}$	6∙51 0∙0164	$\begin{array}{c} 0.012 \\ 0.9 \end{array}$	80.5 72	$36 \cdot 2 \\ 30 \cdot 2$
	_		TABLE 1				

when suitably non-dimensionalized the two sets of data are consistent, as shown by table 1.

The last two columns show two different non-dimensional groups, the first based on wall parameters being the one implicitly suggested by Kline *et al.*, and the second (in which  $\delta^*$  is the displacement thickness) being the one preferred here for reasons to be discussed presently. Considering the errors involved in the measurements, the non-dimensional parameters are in striking agreement, and suggest strongly that the 'bursts' in the hot-wire trace, and in the hydrogenbubble photographs, are related to the same phenomenon.

This simple counting procedure is much more difficult to use at higher Reynolds numbers, as trace (i) in figure 1(b) shows. Even here, however, quiet periods can be consistently identified (as marked on the trace), and we have presented some results so obtained in the following. (A more detailed analysis of such counting procedures will be published separately.) Nevertheless, to eliminate the personal equation involved in simple counting, one has to recognize that the value of any mean burst parameter, including  $\overline{T}$ , will depend on the amplitude discrimination level set to define a 'burst'. It is also necessary to study the effect of the selected frequency pass band of the signal on the measurements.

Figure 2 shows a typical variation of the mean burst rate (i.e. the number of bursts per second) with the amplitude discriminator setting, at various values of the centre-frequency in the pass band. These data were obtained by projecting films of hot-wire traces from a microfilm reader onto graph paper, and counting bursts (by eye) after blocking out central strips of various widths. Periods of activity, i.e. stretches of signal beyond this strip, were counted as separate bursts only if the time interval between them was greater than twice the basic period corresponding to the mid frequency in the selected pass band. It is clear that the burst rate so measured depends in general on the frequency as well as on the discriminator setting. A certain dependence on the discriminator level is of course to be expected (as the burst rate must approach zero for high values of the setting),<sup>†</sup> and is found in intermittency measurements too (e.g. Fiedler & Head 1966). However, to be able to obtain a definite value for  $\overline{T}$  (or, what amounts to the same thing, to identify a 'burst' with certainty), there must be a range of discriminator settings over which the precise value of the setting is immaterial.

<sup>†</sup> The small values of the burst rate in the opposite limit (discriminator setting small) are due to the following. It is only rarely that one notices the absence of a signal at very small discriminator levels; the mean period between such quiet intervals (which must of course be equal to the mean period between bursts) will then be correspondingly long.



FIGURE 2. Values obtained for burst rate at different discrimination levels, using narrow band-pass filtering.  $R_{\theta} = 9450$ . Discriminator level is in arbitrary units; symbols indicate mid-frequency of filter pass band, as follows:  $\blacktriangle$ , 2 kHz;  $\triangle$ , 4 kHz;  $\bigcirc$ , 6 kHz;  $\times$ , 8 kHz;  $\bigcirc$ , 10 kHz.



FIGURE 3. Cross plot of data selected from figure 2 over an optimum range of discriminator levels, showing the burst rate as a function of filter frequency.

From figure 2, there does exist one such range of discriminator levels over which the burst rate is sensibly constant, to within the errors inherent in the measurements; but this range is not as wide as one would wish ! It therefore appears that, while it may not be easily justifiable to speak of a unique value for  $\overline{T}$ , one can nevertheless define a characteristic value for it, corresponding to the maximum burst rate in figure 2. Figure 3 shows a cross plot of the data of figure 2, selected over the range of discriminator settings mentioned above. To within experimental error, it is clear that a reasonably definite characteristic value for the burst period can indeed be defined by the measurements; we will call this value  $\overline{T}_m$ .



FIGURE 4. Burst rate observed when white noise (from commercial random noise generator) is filtered at different frequencies.

Both figures 2 and 3 show that  $\overline{T}_m$  is largely independent of the band-pass frequency provided this frequency is not too low (of course at extremely low frequencies it would not be possible to identify any bursts at all). This independence, we believe, is of fundamental importance; and it is apparently characteristic of turbulent fluctuations. A white-noise signal, processed the same way as the turbulence signal, does also lead to what may be identified as bursts (see figure 1 (b)), but the burst rate then depends linearly on the frequency. Figure 4 shows measurements made on the signal from a commercial random noise generator (VEB Schwingungstechnik Akustik, type NRG 201); an explanation for the linear dependence is offered in the appendix. A similar linear dependence is noticeable in figure 3 for turbulent fluctuations at lower frequencies; the significance of this is discussed in §4.

It turns out that the  $\overline{T}_m$  so determined is directly proportional to the value of  $\overline{T}$  obtained by the simple counting procedure described earlier. Measurements

taken within the inner layer over a wide range of Reynolds numbers are shown in figure 5; for all practical purposes we may take  $\overline{T}$  as being about twice  $\overline{T}_m$ over the entire Reynolds number range covered by the experiments. This fact is perhaps best interpreted as a kind of calibration of the simple counting procedure adopted, and enables us to draw conclusions about the burst rate when either  $\overline{T}$  or  $\overline{T}_m$  is measured.



FIGURE 5. Ratio of  $\overline{T}$ , as obtained by simple counting, to  $\overline{T}_m$ , determined by curves of the type shown in figure 2. The bar in the diagram indicates the uncertainty in the measurements.



FIGURE 6. Distribution across the boundary layer of the burst rate obtained by simple counting. For comparison, the measured mean velocity distribution is also shown (filled circles) in the usual 'law of the wall' form.  $R_{\theta} = 6550$ .

In presenting these results we have occasionally made the tacit assumption that the burst rate does not vary greatly with distance from the wall. The evidence for this is shown in figure 6. Within the margin of error inherent in this type of measurement,  $\overline{T}$  appears to be sensibly constant across most of the boundary layer, and immediately raises the question of its possible relation to the wellknown phenomenon of outer region intermittency.

In a further effort to relate hot-wire and flow-visualization studies, we have also measured the probability distribution of the time interval between bursts T, to be compared with the data of Kim, Kline & Reynolds (1968) presented in the form of histograms. A preliminary report of the present work (Rao, Narasimha & Badri Narayanan 1969) noted how the observed presence of two peaks in all these probability distributions of  $T/\bar{T}$  could not be considered significant, because of the scatter of the measurements. Data now taken with much longer records of



FIGURE 7. The distribution of the interval between bursts, plotted on probability paper. A straight line on this plot corresponds to a log-normal law. Sources:  $\bigcirc$ , present work,  $R_{\theta} = 620$ ;  $\bigcirc$ , Kim *et al.* (1968),  $R_{\theta} = 666$ ;  $\triangle$ , Kim *et al.* (1968),  $R_{\theta} = 1100$ .

hot-wire traces confirm that there is only one peak; furthermore, after trying various standard distributions, it was found that T is best described by a lognormal distribution. How good the fit is can be easily judged from figure 7, which shows the experimental points plotted on appropriate probability paper. For the long record in the present measurements, the fit is excellent; it is also quite good for the data of Kim *et al*. The parameters in the log-normal distributions are, however, different in each case; but there is not yet enough data to relate them to flow parameters.

All available measurements of  $\overline{T}$  (and  $\overline{T}_m$ ) are shown plotted against  $R_{\theta}$  in figure 8, in which  $\overline{T}^+$ ,  $\overline{T}^+_m$  are the burst periods non-dimensionalized using the 'inner' time scale  $\nu/U^2_*$ . The present measurements cover a range of  $R_{\theta}$  from about 600 to 9000. One point taken from the measurements of Tu & Willmarth (1966), made at a large Reynolds number, is also included. In obtaining a numerical value for  $\overline{T}$  from these measurements, we have followed the procedure of Kim *et al.* (1968), and taken the distance between the maxima in the correlation curves as a measure of  $\overline{T}$ . This would be justified if the correlation curve could be modelled, e.g. by the response of a lightly damped harmonic oscillator to a whitenoise forcing function (cf. Batchelor 1953); but there does not seem to be enough data to confirm the validity of the model. Some points from measurements by Laufer & Badri Narayanan (1971) are also included.



FIGURE 8. Burst rate normalized in wall variables as a function of the Reynolds number. Sources:  $\bullet$ , present work;  $\bigcirc$ , Kim *et al.* (1968);  $\times$ , Schraub & Kline (1965);  $\square$ , Runstadler, Kline & Reynolds (1963);  $\triangle$ , Tu & Willmarth (1966);  $\bigtriangledown$ , Laufer & Badri Narayanan (1971). Flagged points indicate present measurements of  $\overline{T}_m$ . The full line is drawn through the data for  $\overline{T}$ , and the dashed line for  $\overline{T}_m$ . Note that the two lines are parallel, indicating same Reynolds number dependence.

The data in figure 8 are completely consistent among themselves, and show (as would be expected) that both  $\overline{T}$  and  $\overline{T}_m$  obey the same scaling laws, except for the numerical factor already discussed. The significant feature in figure 8 is the strong dependence on Reynolds number; the straight line drawn through the data is approximately a  $\frac{3}{4}$ -power law. The possibility of some dependence on the Reynolds number was not ruled out by Kim *et al.*; the range of Reynolds number covered in their experiments was presumably too small to reveal this dependence. It can be argued that any parameter characterized entirely by the inner flow should not show such a strong dependence on Reynolds number when scaled according to the inner variables. Indeed, the correlation shown suggests that  $U_*^2 \overline{T}$  depends only weakly on the viscosity of the fluid, rather like the skin friction coefficient.

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This argument immediately suggests that  $\overline{T}$  scales with parameters describing the outer flow. Several alternative outer scales of length and velocity exist; we have considered the pairs  $(U, \delta^*)$ ,  $(U, \delta)$  and  $(U_*, \delta)$ . Plots of  $\overline{T}$ , using these different scales, are all shown in figure 9, in which the points for  $\overline{T}_m$  are not separately shown, to avoid cluttering the diagram. The parameter  $U\overline{T}/\delta^*$  seems to be independent of  $R_{\theta}$  for all practical purposes; the others do exhibit a small dependence, but the general behaviour and the magnitudes leave no doubt that  $\overline{T}$  is of order  $\delta/U$ .



FIGURE 9. Burst rate normalized with various pairs of outer variables. The straight line on the uppermost plot corresponds to a constant value  $U\overline{T}/\delta^* \simeq 32$ . Symbols have same meaning as in figure 8, except that in the interests of clarity flagged points have been omitted.

## 4. Discussion

The experiments reported here indicate that, although there is some difficulty in clearly identifying 'bursts' in hot-wire traces, a reasonable and definite procedure has been found (using a narrow-band wave analyzer) that leads to quite reliable values of the time interval between bursts. The dependence of the mean burst rate so found on the pass-band centre frequency is interesting: at low frequencies it is approximately linear, as it is in white noise; at higher frequencies, it settles down to a nearly constant value. This is consistent with the view that the smaller eddies have a characteristic structure of their own (Batchelor 1953, ch. 8). As it is known that convection velocities are of the order of U even in the wall layer (e.g. Willmarth & Wooldridge 1962), the fact that  $\overline{T}$  is of order  $\delta/U$  over a wide frequency range suggests that the 'bursts' correspond to regions of concentrated vorticity, which are rich in spectral content and are separated on an average by distances  $L \sim U\overline{T}$  of the order of several boundary-layer thicknesses. Preliminary observations made in turbulence behind a grid and in wakes show similar results, thus strongly supporting the suggestion of Sandborn (1959), that the bursts are related to the well-known phenomenon of intermittency in the small-scale structure of turbulence (observed by Batchelor & Townsend 1949).

It may be recalled that the log-normal distribution, found here for T, was postulated by Kolmogorov (1962), to describe the 'spottiness' of turbulent dissipative structures. Following the usual interpretation of a log-normal law (Kolmogorov 1941), and converting measurements of T to those of distances (as in the preceding paragraph), one may conclude that the rate of change of the separation distance L between the vorticity concentrations is proportional to L, and to some measure of the rate of strain imposed on these structures (see, for example, Saffman 1968).

From the scaling of  $\overline{T}$  and the distribution of T, it therefore appears that the bursts observed in a turbulent boundary layer are related to the small-scale intermittency of Batchelor & Townsend, and the spottiness of Kolmogorov & Landau. In other words, such bursts may well be a fairly general feature of all turbulent flows.

The outer scaling of these motions even in the inner layer suggests, almost by definition, that they correspond to the so-called 'inactive' component (Townsend 1961; Bradshaw 1967). However, this does not mean immediately that they play no role in turbulent energy production in the inner layer; they may still do this by creating regions of intense shear in the inner layer, thus triggering local instabilities. One visualizes large outer eddies scouring the slow-moving inner layer, and releasing bursts of turbulent energy, rather in the manner described by Greenspan & Benney (1963) in their analysis of the non-linear stage in the transition from laminar to turbulent flow. The inner layer would then be neither passive nor solely responsible for the energy production, but would strongly interact with the outer region.

A further indication of this coupling is obtained by a re-examination of the hydrogen-bubble data for the burst rate per unit span F. Kline *et al.* (1967) scale this also on inner variables; and, although the results are reasonable (see points for  $F^+ = F\nu^2 U_*^{-3}$ , figure 10), we know from measurements of  $\overline{T}$  that, over a large Reynolds number range, such scaling is almost certain to be inadequate. A purely outer scaling of the kind found successful here with  $\overline{T}$  suggests  $U/\delta^{*2}F$  as the relevant non-dimensional parameter, but figure 10 shows it depends rather strongly on the Reynolds number. On the other hand, a mixed scaling, using inner variables for the transverse spatial scale and outer for the time, leads to

 $UU_*/\nu\delta^*F$  as the relevant parameter, and this is indeed practically independent of Reynolds number.

An attractive physical model for the flow structure, suggested by these considerations, would be one based on a non-linear stability analysis of the Greenspan-Benney type mentioned above. It is true that this analysis does not provide a specific value for the spanwise period of the structure; but, from a cursory examination of the streamlines calculated by Greenspan & Benney (1963), it appears possible that the transverse structure is connected with the flow at the critical layer, and hence with the inner region, within which the critical layer would be located in the standard turbulent velocity profile.



FIGURE 10. Burst rate per unit span, scaled in different ways. Measurements of Kline *et al.* (1967). +,  $(\frac{1}{4}F^+)$  10<sup>6</sup>;  $\bigcirc$ ,  $U/\delta^{*2}F$ ;  $\bigoplus$ ,  $(UU_*/\delta^{*}\nu F)$  10<sup>-2</sup>.

It will be noted that most of the measurements on which the above arguments are based have been made at not very high Reynolds numbers; the question therefore arises whether the observations are particular to the Reynolds number range covered. In the new experiments reported here, the Reynolds number went up to about  $R_{\theta} = 9000$ , which is beyond the value ( $\simeq 5000$ ) considered by Coles (1962) as the lower limit for the existence of a fully developed turbulent boundary layer. The data-point from Tu & Willmarth is, however, at a truly large Reynolds number ( $R_{\theta} \simeq 4 \times 10^4$ ); it will be noted that, even if the value of  $\overline{T}$  inferred from these measurements is in some error, the main conclusion of the present work regarding the outer scaling of the burst rate will not be affected.

Finally, we note the relevance of these considerations to reverse transition, especially if changes in the burst rate are taken as an indication of the phenomenon. We acknowledge lively discussions and correspondence with P. Bradshaw, G. M. Corcos, S. J. Kline, R. E. Luxton (especially regarding the appendix) and W. W. Willmarth. The Director of the National Aeronautical Laboratory (Bangalore) and his colleagues have materially helped us by a timely loan of some electronic equipment. R. N. thanks Professor I. Prigogine of the Université Libre de Bruxelles for his hospitality during the period when the writing of this paper was completed.

## Appendix

When a random signal is passed through a filter of (small) effective band-width  $\Delta f$  around a centre frequency f, the output of the filter is generally an amplitudemodulated signal of frequency f. The characteristic period of the modulation will clearly be proportional to  $(\Delta f)^{-1}$ , as  $\Delta f$  acts like a beat frequency. This can be seen more formally via the Fourier transform (see, for example, Bracewell 1965): the transform of the output signal will resemble the transfer function of the filter, if the spectrum of the input signal does not vary much over the bandwidth, and will therefore have a maximum at f with sidebands of order  $\Delta f$ . If the resultant modulation is identified with bursts, the mean burst-rate  $1/\overline{T}$  will be proportional to  $\Delta f$ . If, further,  $\Delta f \propto f$ , as it would be in a constant percentage band-width wave analyzer, we may expect that  $\overline{T}^{-1}$  is proportional to f, as measurements show (figure 4). The fact that the burst rate for a turbulent signal is less at high frequencies than would be expected from the above argument (cf. figures 3 and 4) must therefore be attributed to a genuine intermittency in the original signal.

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FIGURE 1. (a) Effect of different ways of processing a signal on the appearance of 'bursts'. All hot-wire traces are those of u' fluctuations in the inner region of a turbulent boundary layer with  $R_{\theta} = 620$ . (i) Unprocessed. (ii) Differentiated. (iii) Filtered with narrow pass band around 800 Hz. (iv) Filtered, with pass band from 800 to 5000 Hz. For illustration of the counting procedure used, each burst is indicated by an arrow.

(b) (i) Turbulent signal, filtered with narrow pass band around 10 kHz;  $R_{\theta} = 9450$ . Quiet periods are now indicated by arrows, question marks being doubtful cases. (ii) White noise, filtered around 6 k Hz.

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